For office use only	Team Control Number	For office use only
T1 ______________	**IMMC 23436160**	F1 ______________
T2 ______________		F2 ______________
T3 ______________	Problem Chosen	F3 ______________
T4 ______________	**D**	F4 ______________

2023

Ice Storage Air Conditioners

Summary

Universally, all companies face a fundamental dilemma: how to reduce costs while extracting the maximal profit possible? In terms of electrical power, ice storage air conditioner, an innovative conditioning technology, is capable of transferring peak hour electrical consumption during the day to valley hours at night, which generates substantial financial savings for shopping mall administrations. To minimize cost while maximizing energy output, we adopt a series of Nonlinear Programming equations to determine an optimized solution and apply our formulas to both LTAS and RTAS ice storage conditioners.

In the problem that the shopping mall administration posed to our team, four main tasks have been accomplished: research on the electrical pricing policy of Shenzhen city, investigating features of conventional and ice storage air conditioners in terms of economics and power usage, and finally calculating the time taken to return investments made switching to ice-storage air conditioners based on a hypothetical scenario of the Yitian Holiday Plaza.

Under the time-of-use pricing policy from ”Industrial and Commercial Electricity Tariff in Shenzhen”, we derive a list of formulas that addresses the various electricity and installation costs, power consumption, and efficiency for ice storage and conventional air conditioners simultaneously. Discussing two types of ice storage air conditioners in parallel (**RTAS** and **LTAS**), we determine: RTAS ice storage conditioners consume **33.1%** more power than conventional conditioners daily while reducing electricity costs by **66.9%**; LTAS ice storage conditioners take in **24.9%** more power compared to conventional conditioners but results in a saving of **69.0%** per day.

Based on the presumption that 20% of all refrigerating capacity is lost to the environment during the course of nightly ice storage, our team use provided requirements as restraints to an optimization problem and use **Nonlinear Programming** with **Genetic Algorithms** to find the optimal points for 4 sets of restraining equations dictating the **setup cost**, **energy consumption**, **investment paid-back time**, and **energy storage** respectively. Summing up the 4 criterion, we implement the **Analytical Hierarchy Process** to devise a final score that is inversely proportional to the total benefit gained from implementing ice storage conditioners. Finally, addressing Task 4, two sets of convincing conclusions are established for investment paid-back time: RTAS ice storage conditioners demands **8.98 years** in paid-back time, while LTAS ice storage conditioners requires **11.02 years** in paid-back time after the initial installation. These two results ultimately suggest that it is highly beneficial for mall administrations to invest in the implementation of ice storage air conditioners due to the appreciable rate at which ice storage generates financial savings.

Finally, we conduct a comprehensive analysis about strength and weakness and conclude that our model is resistant to change and its results are robust and reliable.

Keywords: Nonlinear Programming, Target Functions, Analytical Hierarchy Process, Genetic Algorithm

#IceToWater

Ice Storage Air Conditioner

Source: TEAM 23436160

Science behind Ice Storage Air Conditioner:

The underlying principle of the ice storage air conditioner is: manufacturing ice when the electricity price is low at night, and freezing the ice in insulated containers until conditioning is required in the day when electricity bills are higher. By shifting electrical consumption from "peaks" to "valleys", ice cold storage conditioning succeeds in generating financial savings for shopping malls - whilst achieving the same cooling effects!

Economic Benefits of Ice Cold Air Conditioner

Purchasing electricity during valley hours (predominantly 23:00-7:00) and storing it in ISAC to be used during the operation time of the shopping mall culminates substantial financial savings for mall management. While ISAC is advantageous in lowering electric bills, its startup cost generally stands 3 times as high as that of conventional air conditioning in typical brands such as Midea.

As TEAM 23436160, we perform the meticulous task of calculating the cost benefits for the installation of ISAC in shopping malls. Weighing the savings in reduced electrical bills per month to ISAC's installation and maintenance fees, we conclude that it would ultimately take 8.98 years in paid-back time for RTAS ice storage conditioners and 11.02 years for LTAS ice storage conditioners. (See next section)

Two Types of Ice Cold Storage AC

Two standards have been proposed for Ice Cold Storage AC: Low Temperature Air Supply and Regular Temperature Air Supply.

LTAC excels in lowering power consumption by reducing the temperature of incoming air supply to a range between 4-9 degrees Celsius, so that cooling can take place with less power. On the other hand, RTAS resolves in using conventional air supply methods at a higher temperature for air supply input, thus requiring greater power input. In other aspects, LTAS air conditioners are more environmentally-friendly and can reduce humidity and wind level, creating a more comfortable environment for the users.

Contents

1 Introduction

Air conditioners are widely used in public spaces. Large malls especially have great demands for cooling. Air conditioners consumes significant amounts of power, surmounting to high electrical bills. A solution to saving energy costs is by avoiding peak hours in electrical consumption. Price of electricity is significantly cheaper during night time, where the demand is lower. A proposed solution to save cost utilizes this principle. By creating ice using cheaper electricity during the night, and releasing it to cool air during the day hypothetically lowers costs for energy.

1.1 Problem Restatement

The goal of our research is to construct a model that will calculate the savings in energy costs by switching to an ice storage solution, and how long it will take for investments to be returned from the savings in switching.

2 Assumptions and Notations

2.1 General Assumptions

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

- **Twenty percent of all ice is lost into the surroundings.**
  We assume that there is no extra cost for storing the made ice in low temperature, and thus the consideration of the lost ice needs to be taken. We account the natural rate of ice wastage as 20% of its total refrigerating capacity generated per night.

- **Business hours of all shopping malls begins at 10:00 and ends at 22:00.**
  To approach the calculation of total energy consumption as well as electrical bills, it is vital to maintain a set standard for the operating hours of air conditioning. This time frame is representative of a large majority of shopping malls in China.

- **All air conditioning in this paper runs at full capacity.**
  We assume that air conditioning makes use of the maximum power supply as granted by the product information. This is because it is particularly difficult to impose a model that considers the real operating capacity of air conditioners under influence from a range of external and internal factors.

- **Total cooling demand over a year is constant.**
  Although cooling demand fluctuate base on temperature and traffic, this paper assumes that the total cooling demand over a year is constant.

- **Ice Storage and conventional air conditioners do not require electricity during inactivity**
  For both ice storage and conventional air conditioners, we assume that when the conditioners are not in operation between October and April, no electricity costs and generated and no substantial maintenance fees are required.

- **The total days that the air condition will be on in a year remains constant.** We assume that air conditioners are used on a day-to-day basis from the beginning of April to the end of September, which approximates 182 days.

2.2 Notations

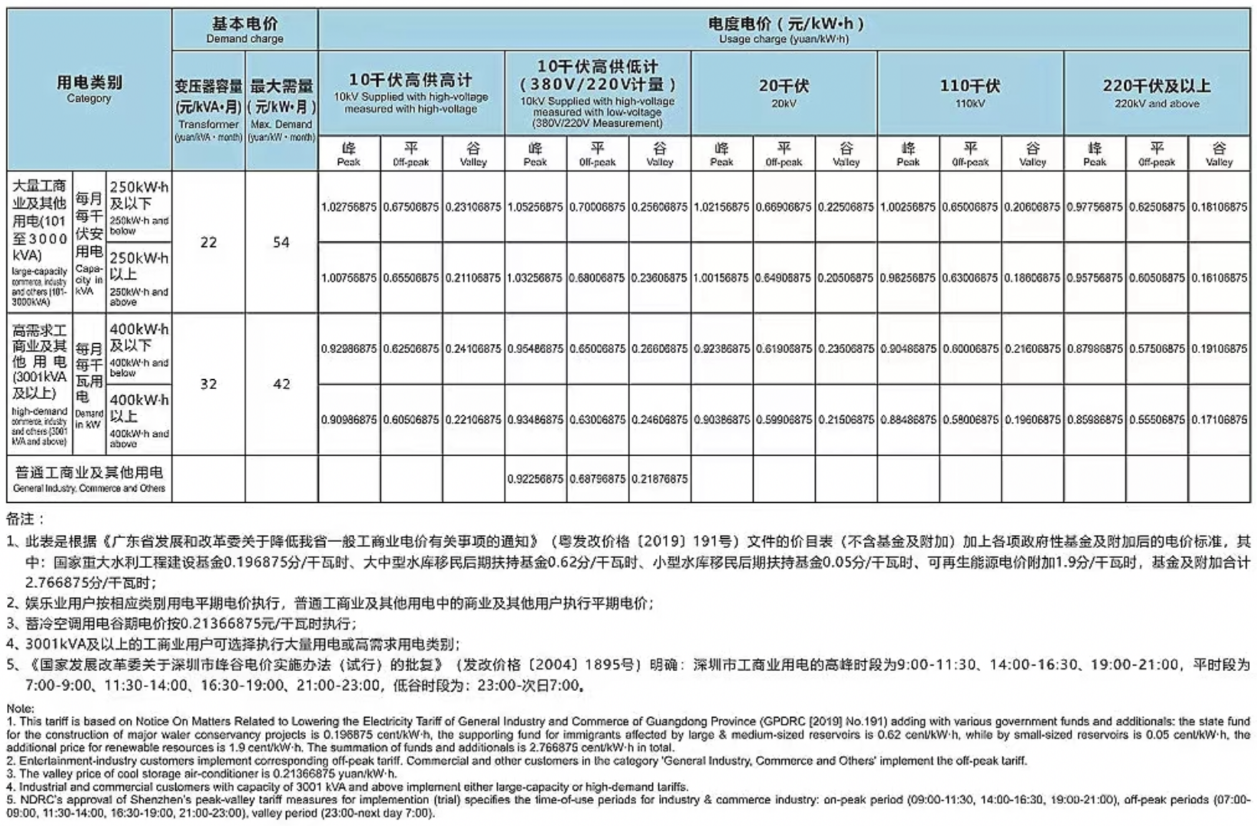
Table 1 Notations and Descriptions

Symbols	Description
C	Total setup cost during installation
$\mathrm{P}_{\mathrm{valley}\,1}$	Price of electricity per kWh during valley hours
$\mathrm{P}_{\mathrm{offpk}}$	Price of electricity per kWh during off-peak hours
$\mathrm{P}_{\mathrm{peak}\,2}$	Price of electricity per kWh during peak hours
PLR_{c_i}	Current unit partial load rate of the i^{th} chiller
q_{s_i}	Cooling capacity of the i^{th} chiller
CL_b	Shopping mall's cooling load
PLR_b	Shopping mall's load rate
N_{c0}	Initial number of conventional air conditioner
N_c	Number of conventional conditioners after the reconstruction of old shopping malls
N_{c_1}	Number of conventional conditioners of new shopping malls
N_{is1}	Number of RTAS-powered ice-storage conditioners
N_{is2}	Number of LTAS-powered ice-storage air conditioners
S_m	Commercial area of the shopping mall
RC_c	Refrigerating capacity (power) of conventional air conditioners
RC_{is1}	Refrigerating capacity (power) of RTAS-powered ice-storage air conditioners
RC_{is2}	Refrigerating capacity (power) of LTAS-powered ice-storage air conditioners
W	Level of ice wastage in the dispersal of heat energy
P_c	Total power consumption of all chiller sets
q_{c_i}	Cooling capacity of the i^{th} chiller
COP_i	Unit performance coefficient under the assumed refrigeration
A, B, C	Constant in the calculation for shopping mall's load rate
E_s	Total energy storage
V_s	Volume of ice stored
C_α	Cost of removing 1 conventional air conditioner
C_β	Cost of installing 1 new air conditioner
C_γ	Cost of purchasing 1 ice-storage air conditioner
C_ω	Cost of purchasing 1 conventional air conditioner
TRC	Total refrigerating capacity (power) needed
TP_c	Daily power consumption of conventional conditioners
TP_{is1}	Daily power consumption of ice storage conditioners with RTAS
TP_{is2}	Daily power consumption of ice storage conditioners with LTAS
t_1	Peak operation hours of conventional conditioner prior reconstruction
t_2	Off-peak operation hours of conventional conditioner prior reconstruction
t_3	Valley operation hours of ice-storage conditioner post reconstruction
t_4	Peak hours of conventional air conditioner post reconstruction
t_5	Off-peak hours of conventional air conditioner post reconstruction
n	Number of ice-storage tanks
C_1	Cost of removing conventional air conditioners
C_2	Cost of installing new air conditioners

**Power consumption and refrigerating capacity can be used interchangeably.*

3 Task 1: Electricity Price Policy

By conducting extensive research regarding the time-of-use electricity price policy in our city Shenzhen, we conclude the pricing standards of electricity at peak, off-peak, and valley time slots in Figure 1.



用电类别 Category			基本电价 Demand charge		电度电价（元/kW·h） Usage charge (yuan/kW·h)														
			变压器容量 (元/kVA·月) Transformer (yuan/kVA · month)	最大需量 (元/kW·月) Max. Demand (yuan/kW · month)	10千伏高供高计 10kV Supplied with high-voltage measured with high-voltage			10千伏高供低计 (380V/220V计量) 10kV Supplied with high-voltage measured with low-voltage (380V/220V Measurement)			20千伏 20kV			110千伏 110kV			220千伏及以上 220kV and above		
					峰 Peak	平 Off-peak	谷 Valley	峰 Peak	平 Off-peak	谷 Valley	峰 Peak	平 Off-peak	谷 Valley	峰 Peak	平 Off-peak	谷 Valley	峰 Peak	平 Off-peak	谷 Valley
大量工商业及其他用电(101至3000kVA) large-capacity commerce, industry and others (101-3000kVA)	每月每千伏安用电 Capacity in kVA	250kW·h及以下 250kW·h and below	22	54	1.02756875	0.67506875	0.23106875	1.05256875	0.70006875	0.25606875	1.02156875	0.66906875	0.22506875	1.00256875	0.65006875	0.20606875	0.97756875	0.62506875	0.18106875
		250kW·h以上 250kW·h and above			1.00756875	0.65506875	0.21106875	1.03256875	0.68006875	0.23606875	1.00156875	0.64906875	0.20506875	0.98256875	0.63006875	0.18606875	0.95756875	0.60506875	0.16106875
高需求工商业及其他用电(3001kVA及以上) high-demand commerce, industry and others (3001 kVA and above)	每月每千瓦用电 Demand in kW	400kW·h及以下 400kW·h and below	32	42	0.92986875	0.62506875	0.24106875	0.95486875	0.65006875	0.26606875	0.92386875	0.61906875	0.23506875	0.90486875	0.60006875	0.21606875	0.87986875	0.57506875	0.19106875
		400kW·h以上 400kW·h and above			0.90986875	0.60506875	0.22106875	0.93486875	0.63006875	0.24606875	0.90386875	0.59906875	0.21506875	0.88486875	0.58006875	0.19606875	0.85986875	0.55506875	0.17106875
普通工商业及其他用电 General Industry, Commerce and Others								0.92256875	0.68796875	0.21876875									

备注：

1、此表是根据《广东省发展和改革委关于降低我省一般工商业电价有关事项的通知》（粤发改价格〔2019〕191号）文件的价目表（不含基金及附加）加上各项政府性基金及附加后的电价标准，其中：国家重大水利工程建设基金0.196875分/千瓦时、大中型水库移民后期扶持基金0.62分/千瓦时、小型水库移民后期扶持基金0.05分/千瓦时、可再生能源电价附加1.9分/千瓦时，基金及附加合计2.766875分/千瓦时；

2、娱乐业用户按相应类别用电平期电价执行，普通工商业及其他用电中的商业及其他用户执行平期电价；

3、蓄冷空调用电谷期电价按0.21366875元/千瓦时执行；

4、3001kVA及以上的工商业用户可选择执行大量用电或高需求用电类别；

5、《国家发展改革委关于深圳市峰谷电价实施办法（试行）的批复》（发改价格〔2004〕1895号）明确：深圳市工商业用电的高峰时段为9:00-11:30、14:00-16:30、19:00-21:00，平时段为7:00-9:00、11:30-14:00、16:30-19:00、21:00-23:00，低谷时段为：23:00-次日7:00。

Note:

1. This tariff is based on Notice On Matters Related to Lowering the Electricity Tariff of General Industry and Commerce of Guangdong Province (GPDRC [2019] No.191) adding with various government funds and additionals: the state fund for the construction of major water conservancy projects is 0.198875 cent/kW·h, the supporting fund for immigrants affected by large & medium-sized reservoirs is 0.62 cent/kW·h, while by small-sized reservoirs is 0.05 cent/kW·h, the additional price for renewable resources is 1.9 cent/kW·h. The summation of funds and additionals is 2.766875 cent/kW·h in total.
2. Entertainment-industry customers implement corresponding off-peak tariff. Commercial and other customers in the category 'General Industry, Commerce and Others' implement the off-peak tariff.
3. The valley price of cool storage air-conditioner is 0.21366875 yuan/kW·h.
4. Industrial and commercial customers with capacity of 3001 kVA and above implement either large-capacity or high-demand tariffs.
5. NDRC's approval of Shenzhen's peak-valley tariff measures for implemention (trial) specifies the time-of-use periods for industry & commerce industry: on-peak period (09:00-11:30, 14:00-16:30, 19:00-21:00), off-peak periods (07:00-09:00, 11:30-14:00, 16:30-19:00, 21:00-23:00), valley period (23:00-next day 7:00).

Figure 1 Industrial and Commercial Electricity Tariff in Shenzhen

In researching for Yitian Holiday Plaza, a local shopping mall located in Shenzhen City, we determine a set of industry standards: the usage charge for business-center shopping malls is 20kV, and the monthly usage of power exceeds 250 kW under the category large capacity commercial area. Ultimately, the time-of-use electricity price policy of Yitian Holiday Plaza is specifically summarized in Table 2.

Table 2 Daily pricing plan for Yitian Holiday Mall based on time periods

Period	Period Type	Cost / $\frac{¥}{\text{kWh}}$
0:00 - 7:00	Valley	0.20508875
7:00 - 9:00	Off-peak	0.64906575
9:00 - 11:30	Peak	1.00156875
11:30 - 14:00	Off-peak	0.64906575
14:00 - 16:30	Peak	1.00156875
16:30 - 19:00	Off-peak	0.64906575
19:00 - 21:00	Peak	1.00156875
21:00 - 23:00	Off-peak	0.64906575
23:00 - 24:00	Valley	0.20508875

4 Task 2: Conventional Air Conditioning

Task 2 instructs us to investigate the power and cost for conventional air conditions in a typical business area shopping mall, along side with an estimated daily air conditioner power consumption and electricity cost in summer.

4.1 Estimated Cost in Setup

4.1.1 Setup Cost per Conventional Air Conditioning

The installation price is defined by summing up the individual costs for all components that make up a conventional air conditioning circulation system. Price specifics are listed in Table 3 as follows.

Table 3 Cost of Air Conditioner Components

Component Name	Power / kW	Cost / 10^4 USD
Screwtype Refrigerator	1108	40
Primary Refrigeration Pump	66	14
Secondary Refrigeration Pump	90	
Water Cooling Pump	120	
Water Cooling Tower	25 25	10.4
Expansion Tank	819.8	0.4
Control System		25
Air Processing Unit		116.7
Air Outlet		5.2
Total	2253.8	211.7

4.1.2 Total Setup Cost

We obtain the number of air conditioning required by simply dividing the maximum coverage area of a single conventional air conditioner by the total area of a shopping mall. The coverage area for each air conditioner is set as a constant of 40,000 square meters with reference to evidence by previous studies. In the example of Yitian Holiday Plaza, approximately 4 air conditioning is required to supply cold air at an appreciable rate.

The setup cost for 4 conventional air conditioners totals **56,761,004 ¥**.

4.2 Estimated Power Consumption (Daily)

From data in Table 2, we attain the power consumption for a single air conditioning, which is 2000.3 KW. Since a typical shopping mall operates for 12 hours per day (10:00 - 22:00), we then estimate the power consumption for all 4 conventional air conditioners:

$$n \times 12 \times RC_c = TP_c \tag{1}$$

Where RC_c denotes the refrigerating capacity/power consumption per air conditioner, n denotes the number of air conditioners in a shopping mall, and TP_c denotes the total power consumption of conventional conditioners on a daily basis.

For Yitian Holiday Plaza, this equation translates to: **108,182 kW** per day.

4.3 Estimated Electricity Cost (Daily)

The total electricity cost for all air conditioners per day is calculated by Equation X by considering the different time slots in which electrical power is charged by peak and off-peak prices.

$$(T_{peak} \times P_{peak} + T_{offpk} \times P_{offpk}) \times RC_c \times n = TC_c \tag{2}$$

Where T_{pk} is the hourly time at which the shopping mall is functioning during the peak hour time frame, P_{peak} is the electricity cost per hour per kW during peak hours, T_{offpk} is the operation time of the mall which coincides with off-peak hours, and P_{offpk} is the electricity cost per hour per kW during off-peak hours. Additionally, RC_c denotes the power per air conditioner, while n denotes the number of air conditioning in a shopping mall.

Proceeding to the cost calculation for Yitian Holiday Plaza, we conclude that the total electricity cost (daily) is **89,285 ¥**. A summary of the calculations above is displayed in Table 4.

Table 4 Daily Electricity Cost for Conventional Air Condition

	Peak Hours	Off-peak Hours
Time	10:00 - 11:30	11:30 - 14:00
	14:00 - 16:30	16:30 - 19:00
	19:00 - 4:00	21:00 - 23:00
Total time / h	6	6
Price / $\frac{¥}{kWh}$	1.00156875	0.64906525
Power Per Air Conditioner / kWh	2253.8	
Number of Air Conditioners	4	
Daily power consumption / kWh	108.182	
Daily electricity cost / ¥	89.285 ¥	

5 Task 3: Ice Storage Air Conditioner

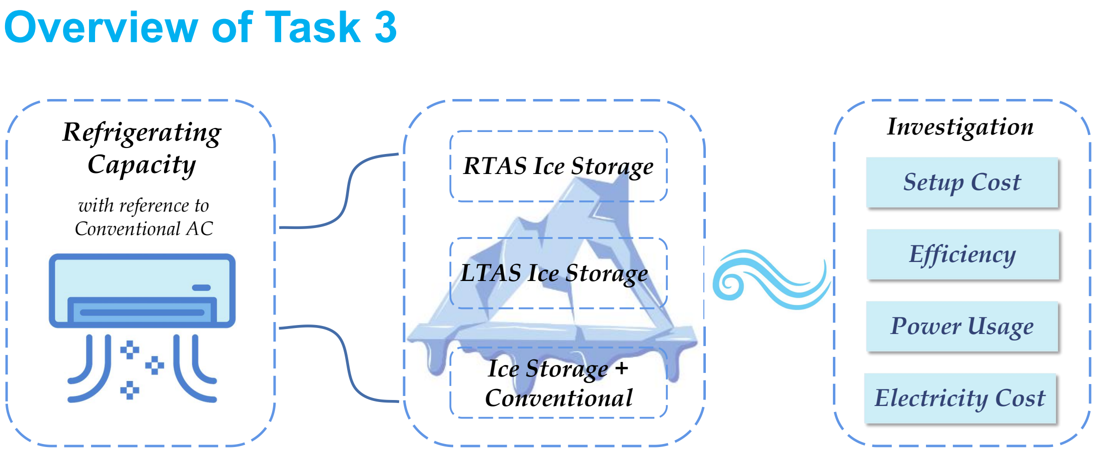


Figure 2 Task 3 Overview

In this section, we consider the total installation cost of ice cold storage air conditioning by summing up the individual costs for each component. To do this, we separate our discussion into two standards of ice storage air conditioning: low temperature air supply (LTAS) and regular temperature air supply (RTAS), each with different installation, maintenance, and electricity costs. We conclude this section by calculating the power consumption, efficiency, and the reduced electrical costs for ice storage air conditioning by comparing the results with those of Task 2.

Low temperature air conditioning makes use of the ice generated in the ice cold storage system and lowers power consumption by reducing the temperature of incoming air supply to a range between 4-9 degrees Celsius. This effectively reduces the volume of air supply required, and yields the same cooling effect with less power. On the other hand, RTAS resolves in using regular temperature air supply methods at a higher temperature for air supply input, thus requiring greater energy to achieve the same cooling effect as LTAS.

Considering the widely different installation costs and power consumption of the two standards given by LTAS and RTAS, it is vital to take separate calculations based on the two criterion.

5.1 Setup Cost

Comparison of Different Air Supply Systems for Ice Storage AC

Device Name	RTAS		LTAS	
	Power / kW	Cost / 10^4 USD	Power / kW	Cost / 10^4 USD
Machine Room Equipment	934.5	129.24	934.5	129.24
Primary Refrigeration Pump	66	13.5	45	12.5
Secondary Refrigeration Pump	90		44	
Water Cooling Pump	90		90	
Air Processing Unit	819.8	116.7	421.7	101.6
VAV Endpoint		-	19.5	
Air Outlet		5.2		13.5
Ice Storage Electrical Savings		-31.02		-35.81
840 ¥/ KVA		-3.60		-8.08
Total	2000.3	230.02	1535.2	232.45

5.1.1 Low Temperature Air Supply

The LTAS installation price is defined by summing up the individual costs for all components that make up an ice cold storage system. By selecting the components which most prominently affects pricing, we obtain the total setup cost for all air conditioning in a shopping mall.

5.1.2 Regular Temperature Air Supply

Likewise, we perform the same calculations to find the initial setup costs for RTAS, as summarized by Table 5. It is worth mentioning that although LTAS exceeds RTAS in terms of its installation and electricity costs, LTAS air conditioners are more environmentally-friendly and can reduce humidity and wind level, creating a more comfortable environment for the users.

5.2 Estimated Power Consumption (Daily)

To calculate the power consumption of ice cold storage air conditioners, we first need to determine the number of air conditioners needed to sustain the refrigerating output as required by the shopping mall. This is calculated through a series of mathematical transformations with respect to the refrigerating capacity of conventional conditioners.

Step 1: Determining the Refrigerating Capacity Generated by Conventional Conditioners

The refrigerating capacity of conventional air conditioners reflects the minimum refrigerating output required by the shopping mall. We perform the simple arithmetic as follows:

$$R_c \times 12 \times n_{c0} = TRC \tag{3}$$

Where R_c is the refrigerating capacity/power of a single conventional conditioner, 12 is the set time for the business hour of a shopping mall, and n_{c0} is the number of conventional air conditioners. Implementing the equation in the circumstances of Yitian Holiday Plaza, we obtain the ultimate refrigerating output required by a shopping mall per day, which equates to **108,182 kW**.

Step 2: Translating Refrigerating Capacity to Number of Ice Storage Conditioners

Obtaining the required refrigerating output, we can then deduce the number of ice storage air conditioning with reference to its power consumption. Specifically, we divide the refrigerating output of the shopping mall by the power consumption (kW) of a single ice storage conditioner and by 8 hours (the assumed operation hour of ice storage conditioners at night) to determine the number of ice storage air conditioners we need to fulfill such refrigerating requirements. Furthermore, subscribing to Assumption 1 that 20% of all ice are lost into the surroundings, we multiply the denominator by a coefficient of 80%.

$$\frac{TRC}{8 \times RC_{is1} \times 80\%} = n_{is1} \qquad for\, RTAS \tag{4}$$

$$\frac{TRC}{8 \times RC_{is2} \times 80\%} = n_{is2} \qquad for\, LTAS \tag{5}$$

Where RC_{is1} represents the refrigerating capacity of RTAC ice storage conditioning, $n_i s1$ represents the number of RTAC conditioning; Similarly, RC_{is2} and n_{is2} denotes the refrigerating capacity and the number of LTAC air conditioners respectively.

Implementing the formulas, we obtain the following results for Yitian Holiday Plaza:

- **Case 1: 9 RTAS-powered** ice storage air conditioners are required to satisfy the mall's minimum refrigerating capacity.
- **Case 2: 11 LTAS-powered** ice storage air conditioners are needed to sustain cooling at an effective rate above the mall's minimum refrigerating capacity.

Step 3: Calculating Power Consumption of Ice Storage Air Conditioning

Upon determining the number of regular air supply and low temperature air supply conditioners respectively, we proceed in calculating their separate power consumption per day.

$$N_{is1} \times 8 \times RC_{is1} = TP_{is1} \qquad for\, RTAS \tag{6}$$

$$N_{is2} \times 8 \times RC_{is2} = TP_{is2} \qquad for\, LTAS \tag{7}$$

According to the above arithmetic, the number of ice cold storage conditioners is multiplied by the 8-hour nightly operation hour and the refrigerating capacity of ice storage conditioners, which is measured in kW.

The following power consumption values are obtained for Yitian Holiday Plaza:

- A daily power consumption of **144,022 kW** is required for powering RTAS conditioning.
- A daily power consumption of **135,098 kW** is required for supplying LTAS conditioning.

Step 4: Calculating Electricity Cost of Ice Storage Air Conditioning

Assuming that ice storage air conditioning functions for 8 hours at full-capacity between 23:00 - 7:00, we can then reference the time-of-use electricity price policy to calculate the electricity costs for both types of ice storage air conditioners respectively.

$$P_{is1} \times 8 \times n_{is1} \times RC_{is1} = C_{is1} \qquad for\, RTAS \tag{8}$$

$$P_{is2} \times 8 \times n_{is2} \times RC_{is2} = C_{is2} \qquad for\, LTAS \tag{9}$$

Given that the electricity cost per kW is 0.20508875 RMB, we are able to conclude the daily cost for ice storage air conditioning by calculating the product for the electricity cost per kW, 8-hour operation time, the number of RTAS/LTAS ice storage conditioners, and the refrigerating capacity for RTAS/LTAS ice storage conditioners. Results are displayed as follows:

- The daily expenditure for RTAS air conditioners amounts to **29,537 ¥**.
- The daily expenditure for KTAS air conditioners amounts to **27,707 ¥**.

Substantial differences can be seen by comparing the power consumption and the electricity costs of ice storage conditioners with that of the conventional air conditioners in Task 2.

Air Conditioners	Daily Power Consumption / kW	Power Consumption Percentage	Daily Electricity Cost / ¥	Electricity Cost Percentage
Conventional Conditioners	108182	-	89,285	-
RTAS Ice Storage Conditioners	144,022	33.1%	29,537	-66.9%
LTAS Ice Storage Conditioners	135,098	24.9 %	27,707	-69.0%

6 Task 4: Non-linear Programming with AHP

To determine the benefits of energy storage, energy saving and cost savings in the shopping malls as well as the years needed to get investment paid back, we establish five non-linear programming models based on AHP.

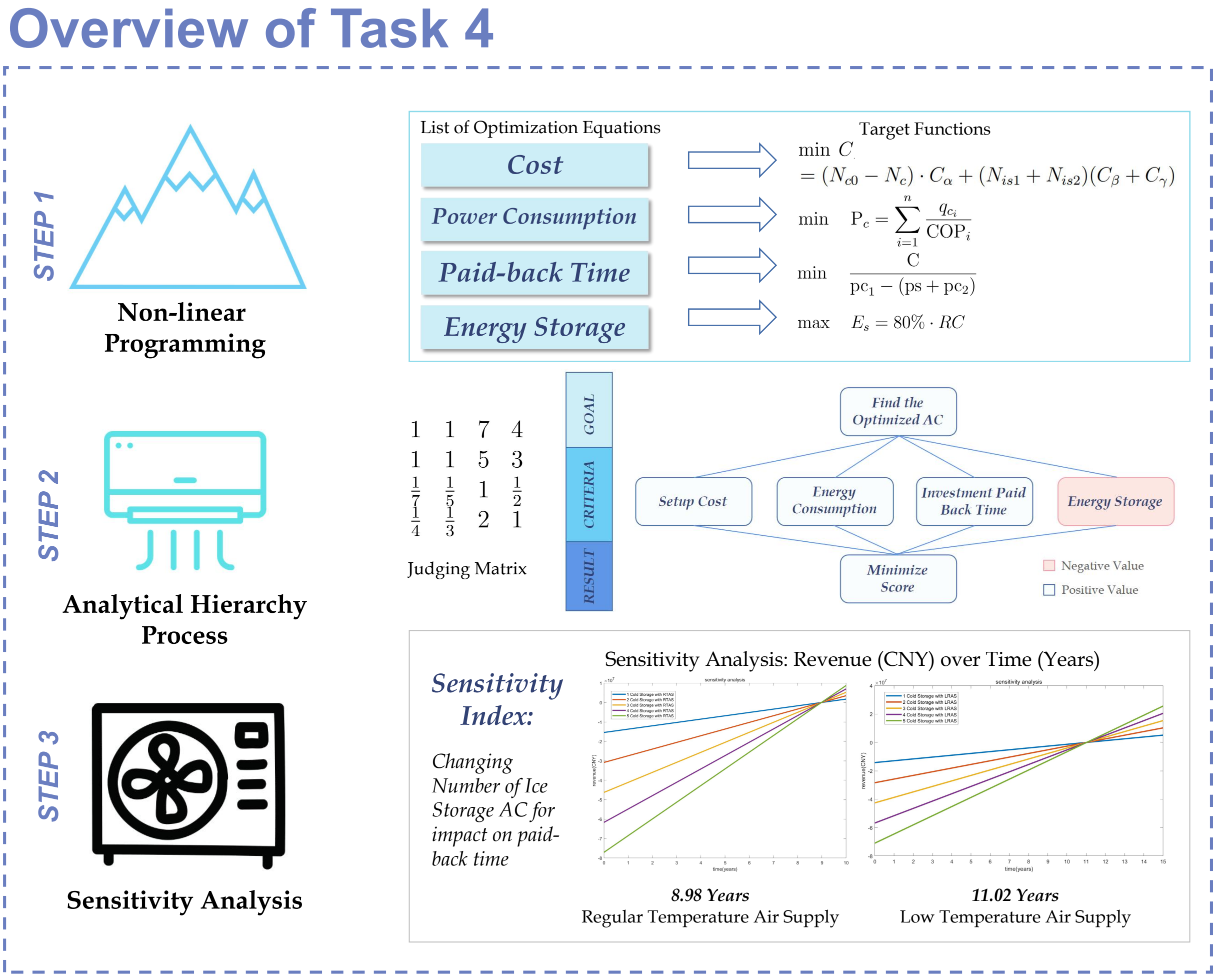


Figure 3 Task 4 Overview

6.1 Non-linear Programming Models on Factors

6.1.1 Setup Cost

The first non-linear programming is used to minimize the cost of investment.

$$
\min \quad C
$$
$$
s.t. \begin{cases} 0 \le \frac{N_c}{N_c+N_{is1}+N_{is2}}, \frac{N_{is1}}{N_c+N_{is1}+N_{is2}}, \frac{N_{is2}}{N_c+N_{is1}+N_{is2}} \le 1 \\ 0 \le RC_c \le 2235.8 \\ 0 \le RC_{is1} \le 2000.3 \\ 0 \le RC_{is2} \le 1535.2 \\ t_3 \cdot (N_{is1} \cdot RC_{is1} + N_{is2} \cdot RC_{is2}) + (t_4 + t_5) \cdot N_c \cdot RC_c \ge TRC \end{cases} \tag{10}
$$

Step 1: Target Function

In the first step, we set the target as min C, which denotes the minimal setup cost; and we derive the expression of C as follows, where C_1 represents the cost related to removing extra conventional air conditioners during the reconstruction and C_2 represents the cost related to the installing of new air conditioners. These two situations are discussed as a better mean of adapting our model to real life scenarios.

$$\min \quad C = C_1 + C_2 = (N_{c0} - N_c) \cdot C_\alpha + (N_{is1} + N_{is2})(C_\beta + C_\gamma) + N_{c_1} \cdot C_\omega \tag{11}$$

Since we consider two cases: the cases in which new air conditioners are installed in newly-built shopping malls and the cases at which conditioners are installed in the reconstruction of old shopping malls, in replacement for conventional air conditioners. These two cases correspond to the two ways in which the following equations are expressed.

- **New shopping malls installing new air conditioners:**
  $N_{c_0} = N_c = 0$,
  $C = (N_{is1} + N_{is2})(C_\beta + C_\gamma) + N_{c_1} \cdot C_\omega$
- **Old shopping malls removing conventional air conditioners as well as installing new air conditioners:**
  $N_{c_1} = 0$,
  $C = C_1 + C_2 = (N_{c0} - N_c) \cdot C_\alpha + (N_{is1} + N_{is2})(C_\beta + C_\gamma)$

Step 2: Constraints

- **Constraint 1: Proportion Constraint**
  We set the number of conventional air conditioners after the replacement as N_C, the number of ice-storage air conditioners with regular temperature air supply as N_{is1}, and the number of ice-storage air conditioners with low-temperature air supply as N_{is2}. Since we aim to explore the proportion of each type of air conditioner in the total number of air conditioners, we need to ensure that the fractional equation can neither be larger than 0 nor smaller than 1. Therefore, we list the first constraint as:

$$0 \leq \frac{N_c}{N_c + N_{is1} + N_{is2}}, \frac{N_{is1}}{N_c + N_{is1} + N_{is2}}, \frac{N_{is2}}{N_c + N_{is1} + N_{is2}} \leq 1 \tag{12}$$

- **Constraint 2: Single Refrigerating Capacity Constraints**
  According to previous studies, various refrigerating capacity limits exist for different types of air conditioners. Hence, we establish numerical ranges that correspond to the different capacity limits of air conditioning. RC_c denotes the refrigerating capacity of conventional conditioners, RC_{is1} denotes the refrigerating capacity of ice-storage air conditioners with regular temperature air supply, and RC_{is2} denotes the refrigerating capacity of ice-storage air conditioners with low-temperature air supply. Thereby, we derive the constraints as follow.

$$0 \leq RC_c \leq 2235.8 \tag{13}$$

$$0 \leq RC_{is1} \leq 2000.3 \tag{14}$$

$$0 \leq RC_{is2} \leq 1535.2 \tag{15}$$

- **Constraint 3: Total Refrigerating Capacity Constraint**
  In total, the refrigerating capacity provided by the air conditioners should be larger than the refrigerating capacity needed in a shopping mall. Hence, by denoting the total refrigerating capacity needed as TRC, we establish the equation that the RC of each air conditioners times their number and time should be larger or equal to TRC.

$$t_3 \cdot (N_{is1} \cdot RC_{is1} + N_{is2} \cdot RC_{is2}) + (t_4 + t_5) \cdot N_c \cdot RC_c \geq TRC \tag{16}$$

6.1.2 Power Consumption

The second non-linear programming is used to minimize the power consumption of all air conditioners.

$$\begin{aligned} \min \quad & \mathrm{P}_c \\ s.t. & \begin{cases} \sum_{i=1}^{n} \mathrm{PLR}_{c_i} \cdot q_{s_i} - \mathrm{CL}_b \cdot \mathrm{PLR}_b \geq 0 \\ 0 < \mathrm{PLR}_{c_i} \leq 1 \end{cases} \end{aligned} \tag{17}$$

Step 1: Target Function

Firstly, we set the target as min P_c, which denotes the power consumption; and we derive the expression of P_c as follows:

$$\min \quad \mathrm{P}_c = \sum_{i=1}^{n} \frac{q_{c_i}}{\mathrm{COP}_i} = \sum_{i=1}^{4} \frac{q_{s_i} \cdot \mathrm{PLR}_{c_i}}{\mathrm{A} \cdot \mathrm{PLR}_{c_i}^2 + \mathrm{B} \cdot \mathrm{PLR}_{c_i} + \mathrm{C}} \tag{18}$$

Step 2: Constraints

- **Constraint 1: Power Consumption Constraint**
  When considering constraints, our first consideration is the power consumption constraint. We need to make sure that the total amount of power generated by the air conditioners is larger or equal to the power consumed in the shopping mall. Thus, we use PLR_{c_i} to represent the current unit partial load rate of the i^{th} chiller, q_{s_i} to represent the rated cooling capacity of the i^{th} chiller, CL_b to represent the building's cooling load, and PLR_b to represent the building's load rate. Hence, we derive the equation that the difference of generated power and consumed power should be larger or equal to zero.

$$\sum_{i=1}^{n} \mathrm{PLR}_{c_i} \cdot q_{s_i} - \mathrm{CL}_b \cdot \mathrm{PLR}_b \geq 0 \tag{19}$$

- **Constraint 2: Partial Load Rate Constraint**
  Since PLR_{c_i} is the current unit partial load rate of the i^{th} chiller, the rate should be designated as larger than zero and smaller than one.

$$0 < \mathrm{PLR}_{c_i} \leq 1 \tag{20}$$

6.1.3 Investment Paid Back Time

The third non-linear programming is used to minimize the investment paid back time of the project.

$$\min \quad \frac{\mathrm{C}}{\mathrm{pc}_1 - (\mathrm{ps} + \mathrm{pc}_2)}$$
$$s.t. \begin{cases} 0 \leq \dfrac{N_c}{N_c + N_{is1} + N_{is2}}, \dfrac{N_{is1}}{N_c + N_{is1} + N_{is2}}, \dfrac{N_{is2}}{N_c + N_{is1} + N_{is2}} \leq 1 \\ 40000 \cdot (N_c + N_{is1} + N_{is2}) \geq S_m \\ t_3 \cdot (N_{is1} \cdot RC_{is1} + N_{is2} \cdot RC_{is2}) + (t_4 + t_5) \cdot N_c \cdot RC_c \geq TRC \end{cases} \tag{21}$$

Step 1: Target Function

At first, we establish the target as min $\frac{\mathrm{C}}{\mathrm{pc}_1 - (\mathrm{ps} + \mathrm{pc}_2)}$, which translates to our optimization objective of minimizing our investment paid-back time. By deduction, we generate the expression of $\frac{\mathrm{C}}{\mathrm{pc}_1 - (\mathrm{ps} + \mathrm{pc}_2)}$ as:

$$\min \quad \frac{\mathrm{C}}{\mathrm{pc}_1 - (\mathrm{ps} + \mathrm{pc}_2)} \tag{22}$$

$$= \frac{C}{RC_c \cdot N_{c_0} \cdot (1.00t_1 + 0.65t_2) - [(RC_{is1} \cdot N_{is1} + RC_{is2} \cdot N_{is2}) \cdot 0.205t_3 + RC_c \cdot N_c \cdot (1.00t_4 + 0.65t_5)]} \tag{23}$$

In the denominator, 1.00 is the price of electricity per hour during the peak time; 0.65 is the price of electricity per hour during off-peak time; and 0.205 is the price of electricity per hour during valley time. Thus, by combining them together, the expression of $\frac{\mathrm{C}}{\mathrm{pc}_1 - (\mathrm{ps} + \mathrm{pc}_2)}$ can be generated.

Step 2: Constraints

- **Constraint 1: Proportion Constraint**

  This constraint is the same as the first constraint in 6.1.2. It is used to constrain the proportion of each type of air conditioner.

$$0 \leq \frac{N_c}{N_c + N_{is1} + N_{is2}}, \frac{N_{is1}}{N_c + N_{is1} + N_{is2}}, \frac{N_{is2}}{N_c + N_{is1} + N_{is2}} \leq 1 \tag{24}$$

- **Constraint 2: Covered Area Constraint**

  Considering the area coverage by the air conditioners, we firstly find the average area that each air conditioner can cover, which is 40000 m^2; by multiplying it with the total number of the three types of air conditioners, we obtain the total area covered, and it should be larger than the total commercial area of the shopping mall, which is S_m.

$$40000 \cdot (N_c + N_{is1} + N_{is2}) \geq S_m \tag{25}$$

- **Constraint 3: Total Refrigerating Capacity**

  The last constraint is the same as the third one in 6.1.2.

$$t_3 \cdot (N_{is1} \cdot RC_{is1} + N_{is2} \cdot RC_{is2}) + (t_4 + t_5) \cdot N_c \cdot RC_c \geq TRC \tag{26}$$

6.1.4 Energy Storage

The fourth non-linear programming is used to maximize the amount of energy storage.

$$\max \quad E_s$$
$$s.t. \begin{cases} 0 \leq PLR_{c_i} \leq 1 \\ 0 \leq E_s \leq V_s \leq V_m \\ 0 \leq RC_c \leq 2253.8 \\ 0 \leq RC_{is1} \leq 2000.3 \\ 0 \leq RC_{is2} \leq 1535.2 \\ t_3 \cdot (N_{is1} \cdot RC_{is1} + N_{is2} \cdot RC_{is2}) + (t_4 + t_5) \cdot N_c \cdot RC_c \geq TRC \end{cases} \tag{27}$$

Step 1: Target Function

The first step is to generate the target function $\min E_s$, which means to minimize the energy storage. By expanding E_s and deriving the expression for it, we establish the following equation.

$$\max \quad E_s = 80\% \cdot RC = 80\% \cdot t_3 \cdot (N_{is1} \cdot RC_{is1} + N_{is2} \cdot RC_{is2}) \tag{28}$$

Step 2: Constraints

- **Constraint 1: Partial Load Rate Constraint** This constraint is the same as the second one in 6.2.2, which is used to limit the rate.

$$0 \leq PLR_{c_i} \leq 1 \tag{29}$$

- **Constraint 2: Volume Constraint** The second constraint is about limiting the volume available for the shopping mall to store the ice and energy. The volume of ice-storing space (V_s) should be larger than zero and smaller than the volume that all ice-storage tanks can hold, which is equivalent to the number of ice-storage tank (n) multiplied by the volume of each ice-storage tank ($63.8m^2$). Hence we generate the equation as follows

$$0 < E_s \leq V_s < 63.8\text{n} \tag{30}$$

- **Constraint 3: Single Refrigerating Capacity Constraints** These three constraints are the same as the second constriants in 6.1.2.

$$0 \leq RC_c \leq 2235.8 \tag{31}$$

$$0 \leq RC_{is1} \leq 2000.3 \tag{32}$$

$$0 \leq RC_{is2} \leq 1535.2 \tag{33}$$

- **Constraint 4: Total Refrigerating Capacity Constraint** The last constraint is the same as the third constraint in Section 6.1.2.

$$t_3 \cdot (N_{is1} \cdot RC_{is1} + N_{is2} \cdot RC_{is2}) + (t_4 + t_5) \cdot N_c \cdot RC_c \geq TR \tag{34}$$

6.2 Analytic Hierarchy Process

After establishing four targets as defined by non-linear programming equations, we need to find a way to combine these criterion together to generate a final optimization. Considering that the criteria of combining should be subjective and leave enough space for shopping mall administrations to dictate their own preferences, we choose to use Analytical Hierarchy Process with the use of Judging Matrix.

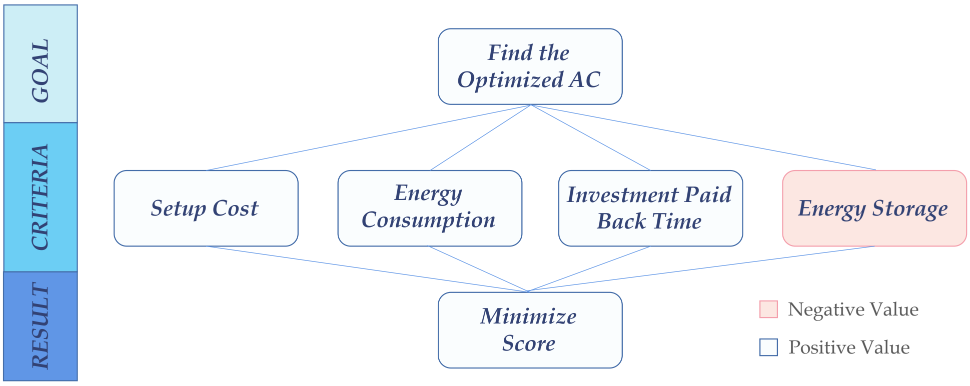


Figure 4 AHP Overview

6.2.1 Goal

We consider the followings factors that we need to optimize in terms of ice storage air conditioners. They constitute: cost, energy consumption, investment paid back time, and energy storage. For comprehensive evaluation, we need to combine these criterion in the following steps.

6.2.2 AHP Process

- **Constructing the Judging Matrix**
  We use the pairwise comparison and one-nine Saaty method to construct the judging matrix $A = (a_{ji})_{4\times4}$ which satisfies $a_{ij} \times a_{ji} = 1$ following the equation(1):

$$\begin{matrix} 1 & 1 & 7 & 4 \\ 1 & 1 & 5 & 3 \\ \frac{1}{7} & \frac{1}{5} & 1 & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{3} & 2 & 1 \end{matrix} \tag{35}$$

- **Calculating Eigenvalues and Eigenvectors**
  The greatest eigenvalue of matrix A is 4.015, and the corresponding eigenvector is $u = (u_1, \ldots, u_n)^T$. Then we normalize u by equation (2):

$$\omega_i = \frac{u_i}{\sum_{j=1}^{n} u_j} \tag{36}$$

- **Conducting consistency check**
  The indicator of the consistency check formula is

$$CI = \frac{\lambda_{\max} - n}{n-1}, CR = \frac{CI}{RI} \tag{37}$$

where n denotes the dimension of the matrix. CR is the expression of the consistency ratio. CI and CR are used in robust analysis, when the value of $CI = 0.005$ less than 0.1 and the value of $CR = 0.005$ less than 0.01, we conclude that the model is dependable.

6.2.3 Result and Discussion

Considering the negative and positive impacts of each criterion, we decide to use mathematical signs to express individual criterion based on whether we want to maximize or minimize the corresponding values. In our AHP model, we maximize energy storage and minimize the rest of the three criterion. Hence, we assign a negative sign for energy storages. Ultimately, the minimized score function, which is also the target of optimization, is shown in Equation 38:

$$\begin{aligned} Total\ Score = {} & 0.375 \times Cost + 0.121 \times Energy\ Consumption + \\ & 0.438 \times Investment\ Paid\ Back\ Time - 0.066 \times Energy\ Storage \end{aligned} \tag{38}$$

6.3 Overall Non-linear Programming Models

After establishing the weight of each factor with the aid of AHP, we derive an equation for the variable $Score$, and our overall goal is to minimize the value of $Score$.

$$\begin{aligned} & \min \quad Score \\ & s.t. \begin{cases} 0 \le \dfrac{N_c}{N_c + N_{is1} + N_{is2}}, \dfrac{N_{is1}}{N_c + N_{is1} + N_{is2}}, \dfrac{N_{is2}}{N_c + N_{is1} + N_{is2}} \le 1 \\ 0 \le RC_c \le 2235.8 \\ 0 \le RC_{is1} \le 2000.3 \\ 0 \le RC_{is2} \le 1535.2 \\ t_3 \cdot (N_{is1} \cdot RC_{is1} + N_{is2} \cdot RC_{is2}) + (t_4 + t_5) \cdot N_c \cdot RC_c \ge TRC \\ \sum_{i=1}^{n} \mathrm{PLR}_{c_i} \cdot q_{s_i} - \mathrm{CL}_b \cdot \mathrm{PLR}_b \ge 0 \\ 0 < \mathrm{PLR}_{c_i} \le 1 \\ 40000 \cdot (N_c + N_{is1} + N_{is2}) \ge S_m \\ 0 \le E_s \le V_s \le 63.8\mathrm{n} \end{cases} \end{aligned} \tag{39}$$

Step 1: Target Function

Minimizing $Score$ will satisfy our final goal which is to find the overall benefit of installing ice storage conditioners. Deducting the expression of $Score$ from the previous steps, we establish our target function as follows:

$$\begin{aligned} \min\ Score = {} & 0.375 \times Cost + 0.121 \times Energy\ Consumption + \\ & 0.438 \times Investment\ Paid\ Back\ Time - 0.066 \times Energy\ Storage \end{aligned} \tag{40}$$

Step 2: Constraints

All 9 constraints included in Equation 39 are explained and evaluated in the previous four sets of non-linear programming equations in Task 4. We obtain the final results by implementing equations with the aid of MATLAB.

6.4 Genetic Algorithm

We use the Genetic Algorithm to solve non-linear programming given the list of constraints.

6.4.1 Flowchart and Pseudocode of Genetic Algorithm

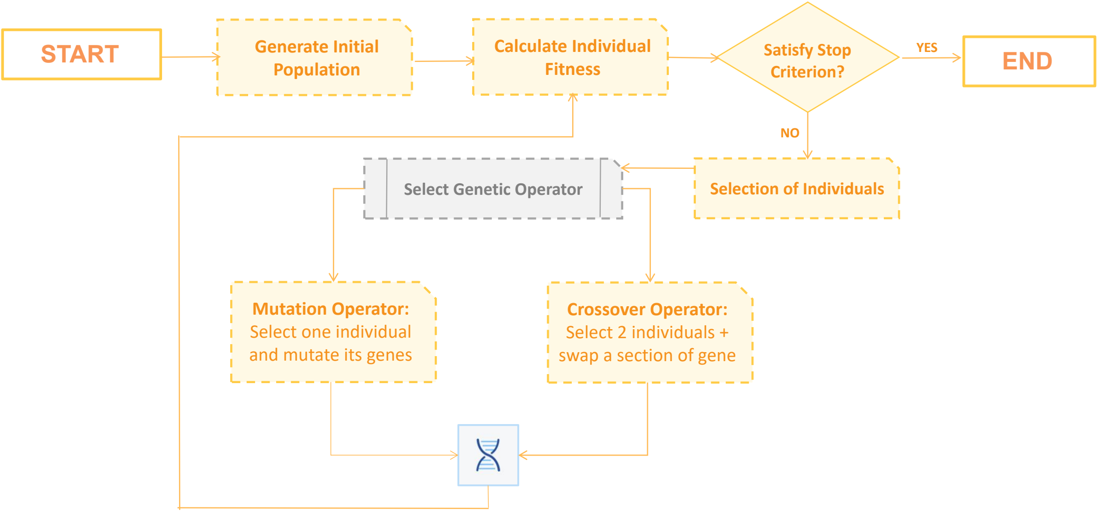


Figure 5 Flowchart of Genetic Algorithm

```
Algorithm 1: Genetic Algorithm
Input: Instance $\Omega$, size $\alpha$ of population, rate $\beta$ of elitism, rate $\gamma$ of mutation, number $\delta$ of
       iterations, the total population $P$
Output: Solution $X$
// Initialization
Generate $\alpha$ feasible solutions randomly, each with 12-parameter long genes;
Save them in the original $P$ ;
for $i \leftarrow 1$ to $\delta$ do
    // Elitism based selection
    $n_e = \alpha * \beta$;
    Select the best $n_e$ solutions in $P$ as $P_1$;
    // Crossover
    $n_c = (\alpha - n_e)/2$ ;
    for $j \leftarrow 1$ to $n_c$ do
        Select $X_A$ and $X_B$ from $P$;
        Generate $X_C$ and $X_D$ from $X_A, X_B$;
        $P_2 \leftarrow \{P_2\} \cup \{X_C, X_D\}$;
    // Mutation
    for $j \leftarrow 1$ to $n_c$ do
        Select $X_j$ from $P_2$;
        Mutate each bit of $X_j$ by rate $\gamma$ and get $X_j'$;
        Check($X_j'$);
        Update $X_j$ with $X_j'$ in $P_2$;
    // Updating
    $P \leftarrow P_1 + P_2$;
return the best solution $X$ in $P$
```

6.4.2 Process of Genetic Algorithm

Genetic Algorithm is divided in to several phases to obtain the final result.

Step 1: Initialization of Population (Coding)

Every gene is represented by a variable in the solution, and the collection of variables that forms the solution is known as the chromosome. Hence, the the population is a collection of chromosomes.

Step 2: Fitness Function

We select the best organism in the species to reproduce offspring out of the available chromosomes, so each chromosome is given a fitness value for ranking. This fitness score enables us to select the individuals who would be selected for reproduction.

Step 3: Selection

Inspired by the Darwinian principle ”Survival of the Fittest”, we establish our main gain to locate the region where more optimal solutions can be obtained, given that there should be a balance between exploration and exploitation of the search space. The Fitness proportionate selection, also known as roulette wheel selection, is used as a genetic operator used in Genetic Algorithms to select potentially useful recombination solutions. The selection process is based on the individual’s fitness function in a given group.

Step 4: Reproduction

The generation of offspring occurs in two ways:

a) Crossover

Crossover is the most vital stage in the Genetic Algorithm, which is acting on behalf of a population group. During crossover, a random point is selected while mating a pair of parents to generate offspring. Then, the new offspring will be added to the population.

b) Mutation

In a few new offspring formed, some of their genes can be subjected to a low random probability mutation. This indicates that some of the bits in the bit chromosome can be flipped. Mutation happens to increase diversity among the population and stop possibilities for premature convergence.

Step 5: Convergence

To decide when to terminate the process, we take into account the following rules:

1) When there is no improvement in the solution quality after completing a certain number of generations set beforehand.

2) When a hard and fast range of generations and time is reached.

3) Until an acceptable solution is obtained.

6.5 Final Result and Discussion

Concluding our research, we establish the following list of findings that reflect the optimal number of air conditioners, the daily cost benefit, and the investment paid-back times. We divide our discussion into two parts, each dictating possible real life combinations.

- **Ice Storage Air Conditioner with RTAS**

We conclude that the number of years for the cost to be paid back is approximately 8.98 years. Each day, ¥9539 is generated in cost savings.

- **Ice Storage Air Conditioner with LTAS**

We conclude that the number of years for the cost to be paid back is approximately 11.02 years. Each day, ¥7152 is generated in cost savings.

7 Sensitivity Analysis

We conducted the sensitivity analysis on our model by changing the number and type of ice storage air conditioners. The results are shown in the following graph.

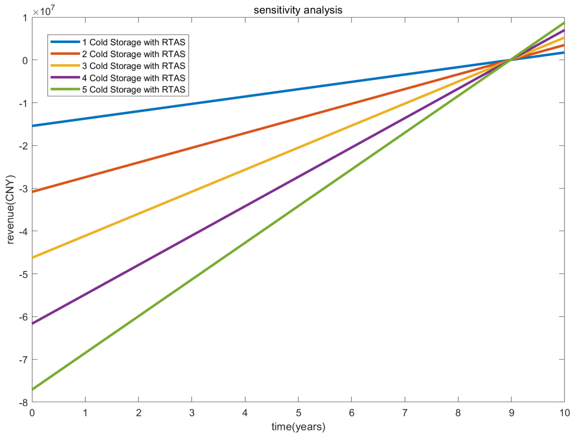


Figure 6 Sensitivity Analysis with Regular Temperature Air Supply

The gradient of the graph, which indicates the economic revenue, is proportional to the number of new air conditioners, and thus the economic revenue of the replacement is positively correlated to the number of new air conditioners purchased.

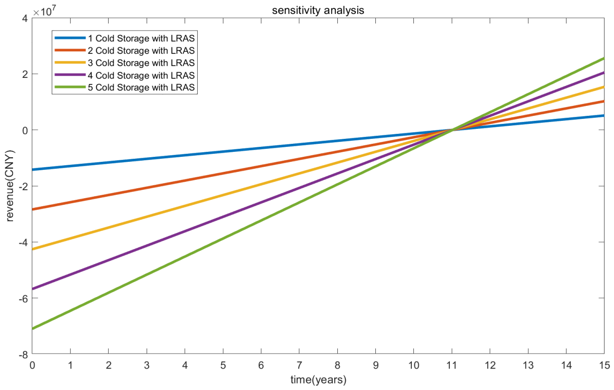


Figure 7 Sensitivity Analysis with Low Temperature Air Supply

By changing the number of ice storage air conditioners, the years for paid back would be constant, as shown in the graph that all curves intersect at the same point which is the time of the cost being completely paid back. This is because each air conditioner purchased could bring a constant reduction in the daily electricity cost.

The optimal number of air conditioner to be purchased, according to the results, is two ice storage air conditioners with convention air supply, at which the time for cost to be paid back is the same while other factors we have also considered are balanced quite well.

8 Conclusion

To conclude, our model incorporates multiple nonlinear equations to solve for the optimal number of air conditioners and the time at which costs could be paid back. Summing up the results derived from optimization, we use the Analytic Hierarchy Process to address the weights of the the four criterion to yield a comprehensive, adaptive result.

8.1 Strengths

- We were able to devise many constraints that fit well using nonlinear programming. Specifically, our adoption of 4 sets of nonlinear programming equations ensures the ”checks and balances” of certain variables - for instance, the maximization of energy storage promotes a increasing number of air conditioners while the minimization of the cost target function encourages a reduced number of air conditioners. This ultimately produces highly accurate predictions that adapt well to real life situations.

- By implementing AHP, not only are we able to seek economic optimization in terms of setup and electricity costs, but we are also environmentally conscious in our consideration for minimum power consumption and maximum power storage.

8.2 Improvements

Despite their superior performance, our model still has areas to improve:

- To better enhance our model’s performance and bolster the accuracy of our results, we should, in the future, use a wider range of data that offers more reliable and convincing conclusions.

- Our model is based on the presumption that ice storage conditioners function for 8 hours at full-capacity during valley electricity hours. However, this setting automatically omits the circumstances where ice storage conditioners function for more/less than 8 hours, depending on fluctuating power demands, which inevitably yields inflexibilities. In future work, rather than treating time as a constant, we could optimize the operating hours for ice storage conditioners as part of our nonlinear equations to find an optimal total score. This would likely improve the applicability of our model in various real world scenarios.

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